# Standard 2D Test Objects for Radiographic Inversion Studies

by
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**Abstract**

This report introduces five two-dimensional standard test objects for radiographic studies. Given a particular experimental design, these objects can be used to create simulated (typically noisy) data. They can then be compared with object reconstructions. The potential applications include testing reconstruction algorithms, examining experimental designs, and probing the limits of existing systems.

Each object is described analytically independent of any finite spatial sampling. Collectively, they encompass a large variety of spatial characteristics and are relevant and applicable to many 2D problems of current interest.

These test objects are not directly applicable to 3D-based reconstruction techniques. Generally, the additional dimension might be spatial or temporal (for example, the use of multiple time shots) or both.

# 1 Introduction

There is a need for cataloging standard objects used for testing radiographic reconstruction algorithms. Such a library provides a standard by which various techniques can be compared and analyzed. This report suggests the use of five simply-defined objects as an initial basis for systematic and principled studies of radiographic techniques.

# 2 The Test Objects

Five test objects have been created for use in radiographic inversion studies. They have been given the names TOn, where n is the integer label of the particular object. TO is an acronym Test Object. The test objects have many properties in common, which we now list.

1. The descriptions are two-dimensional densities defined on the coordinate space $(x, y) \in U \equiv [-1/2, 1/2] \times [-1/2, 1/2]$. The coordinate origin is at the center of each object. The density need not be a mass density. It is used here in a general sense as a density relative to some physical phenomenon (e.g., particle attenuation).

2. Each density description is strictly bounded, $\rho \in [0, 1]$, but the total mass varies from object to object.

3. The objects are described analytically, and are not merely a collection of density values on some predetermined finite lattice.

4. The object mass is concentrated in the central portion of the description space. A clear boundary of zero (or near zero) density surrounds each object.

The five test objects are shown together in Fig. 1. Each subfigure is an approximation with density sampling on a 1024×1024 square voxel grid.

The descriptions will be simplified if we take time now to introduce some notation. Let $I(S)$ be the unit function within the interior of a simply connected closed curve $S$ and zero otherwise. $I$ is an implicit function of $x$ and $y$ over the reconstruction space $U$ so we can simply write

$$I(S) \equiv \left\{ \begin{array}{ll} 1 & \text{if } (x, y) \text{ is interior to } S \\ 0 & \text{otherwise} \end{array} \right\}. \tag{1}$$

Similarly, we define the function $\overline{I}(S)$ to include the boundary of $S$:

$$\overline{I}(S) \equiv \left\{ \begin{array}{ll} 1 & \text{if } (x, y) \text{ is interior to } S \\ 1 & \text{if } (x, y) \in \partial S \\ 0 & \text{otherwise} \end{array} \right\}. \tag{2}$$

These definitions can be further generalized to include sets of curves. We set $I(S_1, S_2, \ldots, S_k)$

equal to unity at some point $(x, y)$ if this point is interior to at least one of the curves $\{S_1, S_2, \ldots, S_n\}$ and to zero otherwise.

Some simple curve definitions will also be useful. We write $C[x, y, r]$ to be the circle of center $(x, y)$ and radius $r$. We also write $P[(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)]$ to be the simply-connected polygon obtained by connecting the ordered coordinate pairs with lines. The final point and first point are also connected.

With these definitions in mind, we consider the specific object descriptions in the following several subsections.

## 2.1 TO1

TO1 is the simplest object. Its density $\rho_1$ is unity within and on the boundary of two nonconcentric circles and is zero otherwise. For the calculation of the object mass, we assume that the defining square region is of unit area (2D volume). The mass is 0.37844. The description of TO1 is

$$\rho_1 \equiv \overline{I}(C[0, 0, 0.375]) - I(C[0.04, 0.04, 0.142]). \tag{3}$$

This test object might be used to examine reconstructions of large regular objects with simple features and sharp boundaries.

## 2.2 TO2

TO2 is a binary density circular object with six overlapping triangular voids.

$$\begin{aligned}
\rho_2 \equiv\ & +\overline{I}( && C[0, 0, 0.375] \ ) \\
& -I( && P[(+0.0780, +0.0194), (-0.3143, -0.0123), (+0.0886, -0.0513)], \\
& && P[(+0.1263, +0.2387), (-0.1584, -0.0270), (-0.0926, +0.1157)], \\
& && P[(+0.2293, +0.0837), (+0.0777, -0.0163), (+0.0841, -0.0961)], \\
& && P[(+0.2659, -0.1508), (-0.2219, +0.0832), (+0.0350, -0.0743)], \\
& && P[(+0.0076, +0.1196), (-0.0157, -0.0312), (+0.0403, +0.1636)], \\
& && P[(+0.1031, -0.2746), (-0.1871, -0.2422), (-0.0856, -0.2003)] \quad ).
\end{aligned} \tag{4}$$

Because the voids are triangular, the collection has spatial features at all length scales less than about the radius of the bounding circle. The mass of TO2 is about 0.39549 and the void fraction is 10.5%. This object might be used to consider material damage models and the ability of reconstruction algorithms to reconstruct complex void patterns.

## 2.3 TO3

TO3 is a discrete density object with circular boundaries. It is formed by summing four circles of density 0.25 and then imposing a zero-density fifth circular region on this sum.

$$\begin{aligned} 4\rho_3 \equiv\ & \{\ \overline{I}(C[0.395, 0.316, 0.201]\ ) \\ & \overline{I}(C[0.502, 0.421, 0.294]\ ) \\ & \overline{I}(C[0.620, 0.369, 0.128]\ ) \\ & \overline{I}(C[0.511, 0.476, 0.134]\ )\ \} \\ & \times\{\ 1 - \overline{I}(C[0.440, 0.560, 0.150]\ )\ \}. \end{aligned} \tag{5}$$

Notice the factor of 4 on the left-hand side of this equation. The mass of TO3 is 0.09635. The densities at all points fall within a discrete set of values:

$$\rho_3(x, y) \in \{0, 0.25, 0.50, 0.75, 1\}\ \forall\ (x, y) \in U. \tag{6}$$

This object is generally composed of large features with sharp edges. The exceptions are the intersections of boundaries that exhibit arbitrarily small spatial characteristics. TO3 is useful for testing the ability to distinguish between different density materials and to locate boundaries.

## 2.4 TO4

TO4 has a smoothly-varying density with values ranging from zero to unity. It is defined by the equation:

$$\rho_4 = sin^2\left[exp\left(-\frac{\overline{x}^2 + \overline{y}^2}{5}\right) \times \left(\overline{x}^3 + \overline{x}^2\overline{y} + \overline{x}\overline{y} - \overline{y}^2 - \frac{1}{\overline{x} - \overline{y} + 312/23}\right)\right], \tag{7}$$

where $\overline{x} \equiv 15x$ and $\overline{y} \equiv 15y$. This function is smooth everywhere on $U$ and tends toward zero for large $x$ and $y$. The squared sine envelope ensures that $\rho_4 \in [0, 1]$. The last fraction of Eq. 7 is never singular over $U$. It is important to note that the density is never identically zero at the periphery of $U$. However, the Gaussian term ensures that the value is very small. The mass of TO4 is 0.19678. This object can be used to test reconstructions of compressible objects.

## 2.5 TO5

TO5 is simply the average of TO3 and TO4:

$$\rho_5 = (\rho_3 + \rho_4)/2. \tag{8}$$

This object is the most complex of the five. TO5 has a smoothly varying density that is interrupted by sharp circular boundaries. Its mass is 0.14657. It can be used, for example, to test reconstructability of shocks in compressible materials.

# 3 Conclusion

We have created a radiography test-object library with the introduction of five 2D objects. These objects are defined analytically independent of experimental designs and reconstruction algorithm details. These objects can be used for generating simulated data, testing reconstruction methods, and examining experiment design parameters. This library is intended to be an ongoing project evolving according to need. Potential additions include 2D objects of greater spatial complexity, 2D object series for simulated dynamics, and 3D objects.

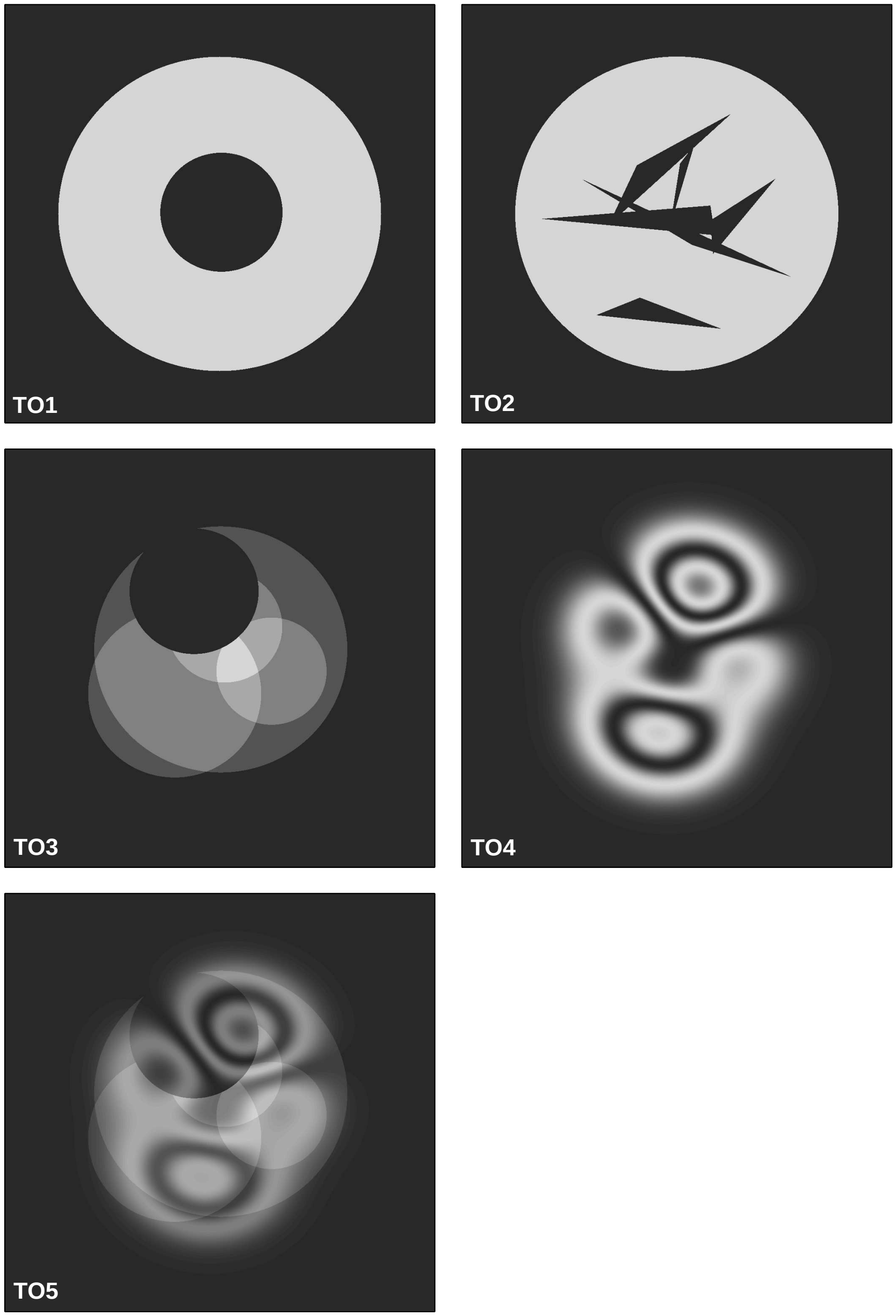


Figure 1: Finitely sampled representations of the five standard test objects.